

11-5 Square Roots of Variable Expressions

Objective: To find square roots of variable expressions and to use them to solve equations and problems.

Property

Property of Square Roots of Equal Numbers For any real numbers r and s :
 $r^2 = s^2$ if and only if $r = s$ or $r = -s$.

CAUTION When you are finding the principal square root of a variable expression, you must be careful to use absolute value signs when needed to ensure that your answer is positive. For example, $\sqrt{x^2} = |x|$, not x .

Example 1 Simplify: a. $\sqrt{144x^2}$ b. $\sqrt{25n^8}$ c. $\sqrt{12a^3}$

Solution a. $\sqrt{144x^2} = \sqrt{144} \cdot \sqrt{x^2}$
 $= 12|x|$

b. $\sqrt{25n^8} = \sqrt{25} \cdot \sqrt{n^8}$
 $= \sqrt{25} \cdot \sqrt{(n^4)^2}$
 $= 5n^4$ (n^4 is always nonnegative)

c. $\sqrt{12a^3} = \sqrt{4 \cdot 3 \cdot a^2 \cdot a}$
 $= \sqrt{4} \cdot \sqrt{a^2} \cdot \sqrt{3} \cdot \sqrt{a}$
 $= 2|a|\sqrt{3a}$

Simplify.

1. $\sqrt{81x^2}$

2. $\sqrt{121x^2}$

3. $\sqrt{20x^2}$

4. $\sqrt{45x^4}$

5. $-\sqrt{25x^2}$

6. $-\sqrt{16c^4}$

7. $-\sqrt{64d^8}$

8. $-\sqrt{98n^6}$

9. $\sqrt{225y^4}$

10. $\sqrt{400a^6b^4}$

11. $\sqrt{81m^{12}}$

12. $\sqrt{441n^6}$

13. $\pm\sqrt{75x^2y^3}$

14. $\pm\sqrt{60x^6y^4}$

15. $-\sqrt{121x^2y^2}$

16. $-\sqrt{900a^4b^6}$

17. $\pm\sqrt{\frac{81x^8}{100}}$

18. $\pm\sqrt{\frac{121}{225x^{10}}}$

19. $\sqrt{\frac{x^4y^8}{9z^2}}$

20. $\sqrt{\frac{32m^3n^2}{2mn^2}}$

21. $\sqrt{\frac{16x^{18}}{3600y^{20}}}$

22. $\sqrt{\frac{256x^{40}}{25}}$

23. $\sqrt{2.25x^4}$

24. $-\sqrt{2.56k^2}$

11-5 Square Roots of Variable Expressions (continued)

Example 2 Simplify $\sqrt{m^2 - 8m + 16}$.

Solution $\sqrt{m^2 - 8m + 16} = \sqrt{(m - 4)^2} = |m - 4|$

Simplify.

25. $\sqrt{x^2 + 4x + 4}$

26. $\sqrt{n^2 - 14n + 49}$

27. $\sqrt{x^2 - 6x + 9}$

28. $\sqrt{m^2 - 10m + 25}$

Example 3 Solve $4x^2 = 25$.

Solution 1

$$\begin{aligned} 4x^2 &= 25 \\ 4x^2 - 25 &= 0 \\ (2x + 5)(2x - 5) &= 0 \\ 2x &= -5 \quad \text{or} \quad 2x = 5 \\ x &= -\frac{5}{2} \quad \text{or} \quad x = \frac{5}{2} \end{aligned}$$

$$\begin{aligned} \text{Check: } 4\left(\frac{5}{2}\right)^2 &\stackrel{?}{=} 25 & \text{and} & & 4\left(-\frac{5}{2}\right)^2 &\stackrel{?}{=} 25 \\ 25 &= 25 \checkmark & \text{and} & & 25 &= 25 \checkmark \end{aligned}$$

The solution set is $\left\{\frac{5}{2}, -\frac{5}{2}\right\}$.

Solution 2

$$\begin{aligned} 4x^2 &= 25 \\ x^2 &= \frac{25}{4} \\ x &= \pm\sqrt{\frac{25}{4}} \\ x &= \pm\frac{5}{2} \end{aligned}$$

Solve.

29. $x^2 = 16$

30. $n^2 = 36$

31. $x^2 - 9 = 0$

32. $d^2 - 25 = 0$

33. $0 = a^2 - 49$

34. $0 = m^2 - 64$

35. $2m^2 - 18 = 0$

36. $40b^2 - 160 = 0$

37. $36y^2 - 16 = 0$

38. $4c^2 - 25 = 0$

39. $0 = 49z^2 - 9$

40. $0 = 45x^2 - 125$

Mixed Review Exercises

Simplify.

1. $\pm\sqrt{80}$

2. $-4\sqrt{75}$

3. $3\sqrt{256}$

4. $2^{-3} - 3^{-2}$

5. $4^3 \cdot 2^{-5}$

6. $(3x^2)^3(-2x^4)^2$

Evaluate if $x = 9$, $y = 16$, and $n = 1$.

7. $x^2 + y^2$

8. x^2n^2

9. $y^2 - x^2$

10. $\sqrt{\frac{y}{n}}$

11. $\sqrt{\frac{x}{y}}$

12. $(\sqrt{y})^2$